Vittorio Casella

DIET – Università di Pavia

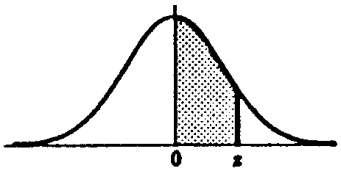
email: vittorio.casella@unipv.it

Calcolo della probabilità sulla normale con le tabelle

Dispense

[tabella_funzione_distribuzione_normale_rot.png]

Tab. 2 - Aree sotto la curva normale standardizzata da 0 a z.



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0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
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1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
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2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
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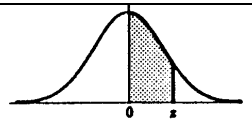
La funzione tabellata

Tab. 2 - Aree sotto la curva normale standardizzata da 0 a z.



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Viene tabellata la funzione

$$F'(x) = \int_0^x \mathrm{d}u\, f_Z(u) \qquad 0 \le x \le 4$$

con passo 0.01. L'apice non ha alcuna relazione con la derivata.

La funzione F' rappresenta evidentemente la probabilità dell'intervallo $[0,x]$

$$F'(x) = P\left(Z \in [0,x]\right) \qquad x > 0$$

Per gli intervalli aventi come estremo sinistro un numero negativo $-x$ vale

$$P\left(Z \in [-x,0]\right) = F'(x)$$

come si dimostra in seguito.

[tabella_funzione_distribuzione_normale.png]

La funzione tabellata – 2

Per gli intervalli aventi come estremo sinistro un numero negativo $-x$ si ha

$$P\left(Z \in [-x,0]\right) = \int_{-x}^{0} \mathrm{d}u\, f_Z(u)$$

Effettuando la sostituzione

$v = -u \qquad dv = -du \qquad$ si ottiene

$$P\left(Z \in [-x,0]\right) = -\int_{x}^{0} \mathrm{d}v\, f_Z(-v) =$$

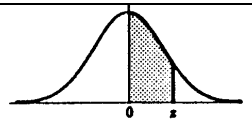
$$= \int_{0}^{x} \mathrm{d}v\, f_Z(-v) = \qquad \text{per le proprietà dell'integrale}$$

$$= \int_{0}^{x} \mathrm{d}v\, f_Z(v) \qquad \text{perchè } f_Z \text{ è simmetrica}$$

$$= F'(x) \qquad \text{CVD}$$

Calcolare la probabilità di un intervallo

Tab. 2 - Aree sotto la curva normale standardizzata da 0 a z.



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2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
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2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
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3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Dato un certo valore x_0, compreso fra 0 e 4 e avente non più di due cifre dopo la virgola, ci si muove sulle righe e sulle colonne fino ad individuare il valore p corrispondente.

Esempi

$$F'(0.43) = 0.1664$$

$$F'(1.40) = 0.4192$$

Che succede se bisogna valutare F' su un x che non compare nella tabella?

Bisogna interpolare.

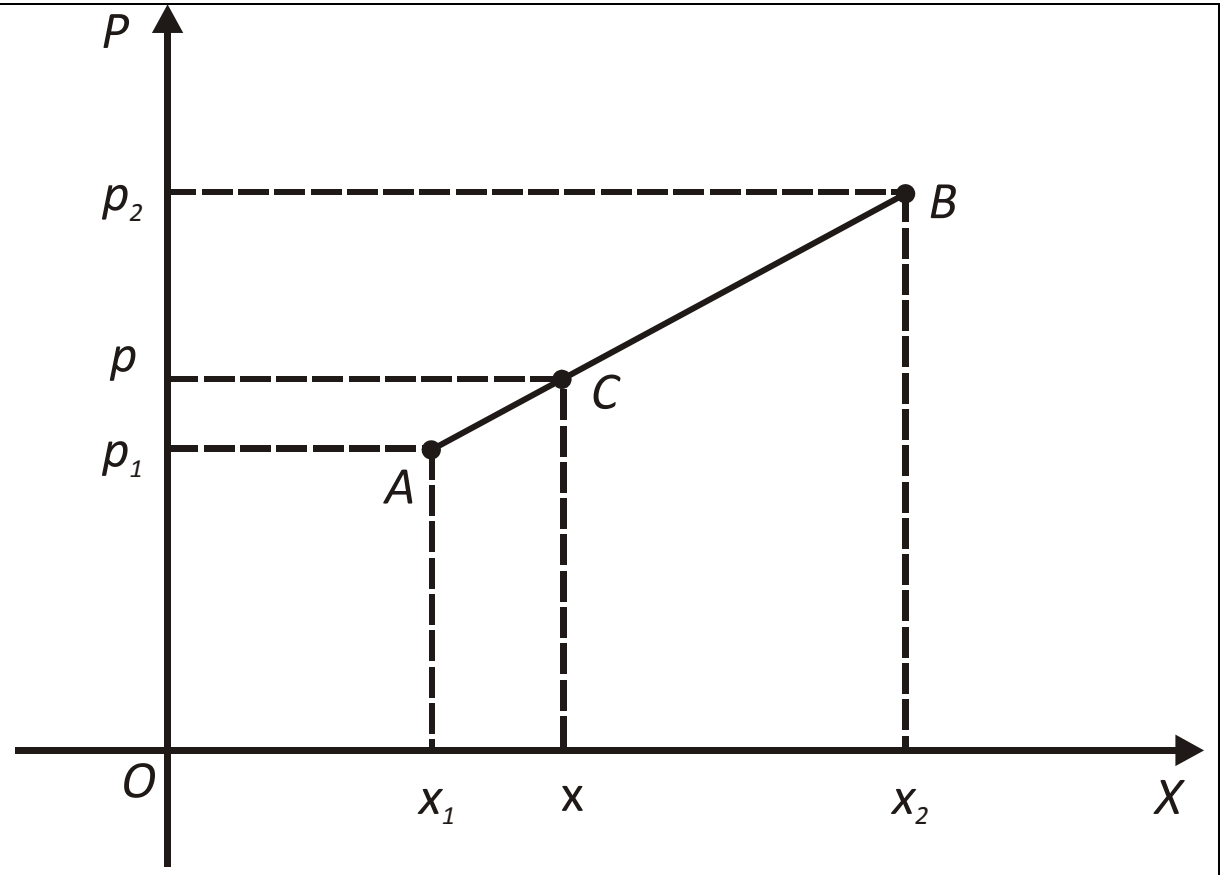
Come interpolare i dati della tabella

La tabella descrive la funzione F' su un insieme limitato di coppie (x,p). Il comportamento della funzione altrove può essere ricostruito, almeno parzialmente, mediante interpolazione lineare

I punti A e B rappresentano due coppie (x,p) presenti in tabella.

Il punto C rappresenta invece una coppia non presente in tabella, da determinare per interpolazione.

La scelta dell'interpolazione lineare equivale all'assunzione che C appartenga al segmento $\overline{AB}$



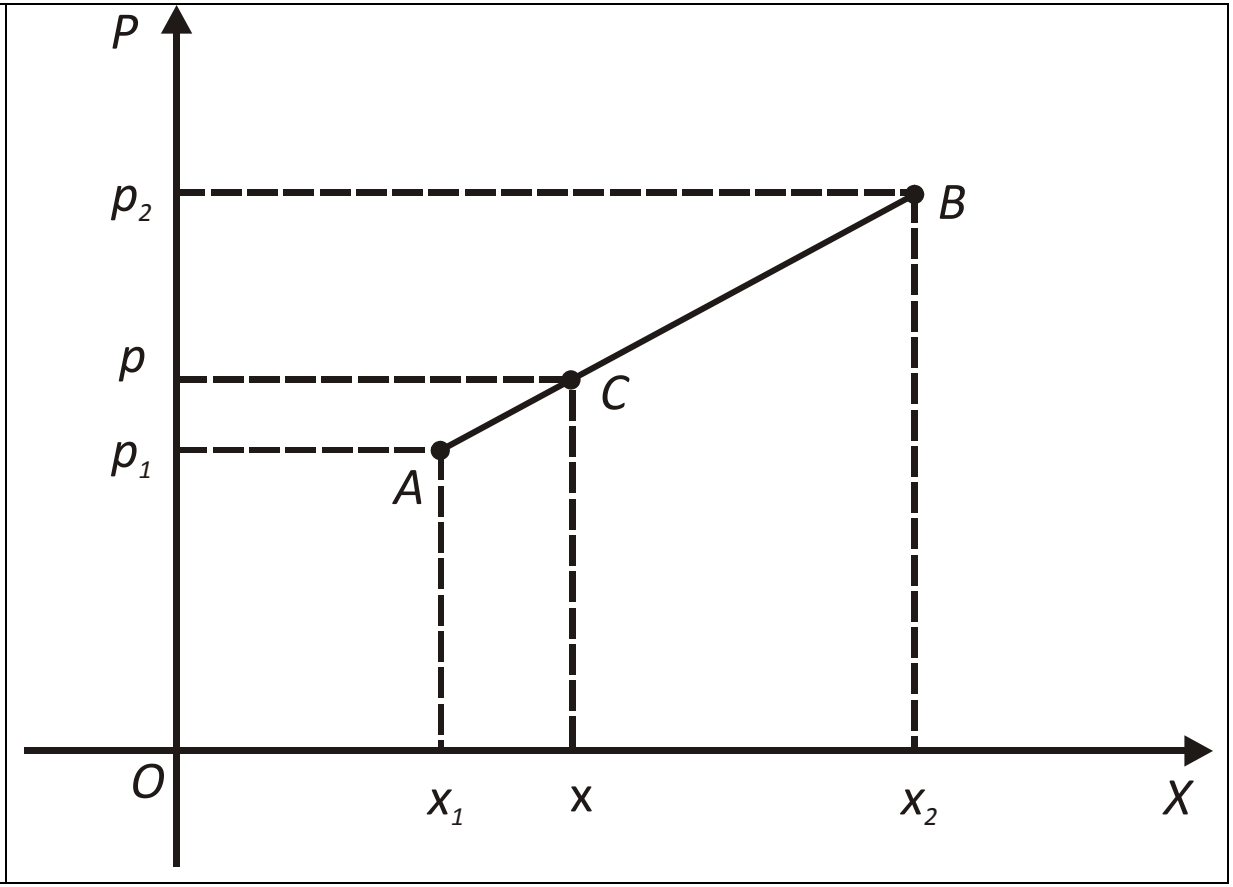
[interpolazione_lineare.cdr;wmf]

Come interpolare i dati della tabella - 2

Il punto C appartiene al segmento $\overline{AB}$.

Dunque i segmenti $\overline{AB}$ e $\overline{AC}$ appartengono alla stessa retta da cui

$$\frac{p - p_1}{x - x_1} = \frac{p_2 - p_1}{x_2 - x_1}$$



Come interpolare i dati della tabella – 3

La relazione

$$\frac{p - p_1}{x - x_1} = \frac{p_2 - p_1}{x_2 - x_1}$$

può essere usata in due modi.

Se si conosce x_0 e si vuole ricavare il corrispondente p

$$p = p_1 + \frac{p_2 - p_1}{x_2 - x_1}\left(x_0 - x_1\right)$$

Se invece si conosce p_0 e si vuole ricavare il corrispondente x

$$x = x_1 + \frac{p_0 - p_1}{p_2 - p_1}\left(x_2 - x_1\right)$$

Calcolare la probabilità di un intervallo mediante interpolazione

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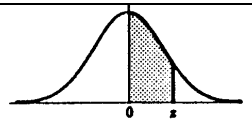

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1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Cerchiamo la probabilità dell'intervallo

$$P\{[0,0.903]\}$$

Si ha

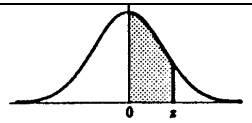
$$p = p_1 + \frac{p_2 - p_1}{x_2 - x_1}(x_0 - x_1) =$$

$$= 0.3159 + \frac{0.3186 - 0.3159}{0.91 - 0.90}(0.903 - 0.900) =$$

$$= 0.3167$$

Trovare l'intervallo avente una certa probabilità

Tab. 2 - Aree sotto la curva normale standardizzata da 0 a z.



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Dato un valore di probabilità p_0 compreso fra 0 e 0.5, avente non più di 4 cifre decimali, lo si cerca all'interno della tabella e poi ci si sposta sulla cornice per determinare riga e colonna corrispondenti, da cui risalire a x.

Cerchiamo ad esempio il valore di x che soddisfa $F'(0.475)$. Si ha

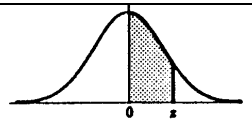
$$x = 1.96$$

Cerchiamo ora il valore di x che soddisfa $F'(0.495)$. Si deve interpolare ottenendo

$$x = 2.575$$

Trovare l'intervallo avente una certa probabilità - 2

Tab. 2 - Aree sotto la curva normale standardizzata da 0 a z.



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Cerchiamo di risolvere il problema per

$$p_0 = 0.4230$$

Esso cade fra

$$p_1 = 0.4222 \text{ e } p_2 = 0.4236$$

Il corrispondente valore x deve essere determinato per interpolazione lineare.

Partendo da

$$x = x_1 + \frac{p_0 - p_1}{p_2 - p_1}\left(x_2 - x_1\right) =$$

$$= 1.42 + \frac{0.4230 - 0.4222}{0.4236 - 0.4222}\left(1.43 - 1.42\right) =$$

$$= 1.4257$$

La probabilità di intervalli che cadono nel semiasse positivo

Dovendo calcolare

$$P\left(\left[a,b\right]\right) \qquad a,b>0$$

Banalmente

$$P\left(\left[a,b\right]\right)=F'(b)-F'(a)$$

La probabilità di intervalli a cavallo dell'origine

Per calcolare $P([a,b])$ con $a<0$ e $b>0$ si ha

$$P([a,b]) = P([a,0]) + P([0,b])$$

Per motivi di simmetria

$$P([a,0]) = P([0,-a])$$

da cui

$$P([a,b]) = P([0,-a]) + P([0,b]) =$$
$$= F'(-a) + F'(b)$$

La probabilità di intervalli che cadono nel semiasse negativo

$$P([a,b]) \qquad a,b<0$$

Per motivi di simmetria si ha

$$P([a,b]) = P([-b,-a]) \text{ con } -a,-b>0$$

Di conseguenza

$$P([a,b]) = P([-b,-a]) = F'(-a) - F'(-b)$$

Esercizio 1

Calcolare la probabilità dell'intervallo $[3,6]$ per la $N[4,1]$.

Standardizziamo anzitutto

$$[a,b]=[3,6]$$

$$[a',b']=\left[\frac{a-\mu}{\sigma},\frac{b-\mu}{\sigma}\right]=[-1,2]$$

Dunque

$$P\{N[4,1]\in[3,6]\}=P\{Z\in[-1,2]\}=F'(2)+F'(1)=$$

$$=0.4772+0.3413=0.8185$$

Matlab

```
normcdf(6,4,1)-normcdf(3,4,1)
```

```
=0.8186
```

La differenza? Un leggero disaccordo fra i dati della tabella e i conti di Matlab.

Penso abbia ragione quest'ultimo.

Esercizio 2

Calcolare la probabilità dell'intervallo $[3.63, 4.02]$ per la $N[4.12, 0.34]$.

$$[a', b'] = [-1.4412, -0.2941] = F'(1.4412) - F'(0.2941) =$$

$$= 0.3096$$

NB: possibili variazioni di 1/2 unità sull'ultima cifra decimale

Calcolo dell'intervallo di confidenza simmetrico per la Z

Dato un livello di significatività α (in genere 1% o 5%) l'intervallo di confidenza simmetrico è simmetrico rispetto all'origine (la media della Z) ed ha probabilità $1-\alpha$.

I suoi estremi vengono generalmente indicati con

$$Z_{\alpha/2} \text{ e } Z_{1-\alpha/2}$$

dove la notazione significa

$$F_Z\left(Z_{\alpha/2}\right)=\alpha/2$$

$$F_Z\left(Z_{1-\alpha/2}\right)=1-\alpha/2$$

Vale evidentemente, per la simmetria della Z,

$$Z_{\alpha/2}=-Z_{1-\alpha/2}$$

Calcolo dell'intervallo di confidenza simmetrico per la *Z* - 2

Calcoliamo ad esempio l'intervallo di confidenza per $\alpha = 5\%$.

Cerchiamo $Z_{0.975}$. Usando la tabella, dobbiamo cercare la x corrispondente a

$p_0 = 0.475$

$x = 1.96$

Da cui

$IC\{Z, 5\%\} = [-1.96, 1.96]$

Matlab

```
norminv(0.975,0,1)
```

Calcolo dell'intervallo di confidenza simmetrico per la normale

Calcoliamo l'intervallo di confidenza al livello $\alpha = 4\%$ della $N[10,2]$.

Determiniamo anzitutto l'intervallo di confidenza allo stesso livello di significatività per la Z:

$$Z_{0.98} = 2.0537$$

Nel linguaggio della standardizzazione

$$[a',b'] = [-2.0537, 2.0537]$$

da cui

$$[a,b] = [a'\sigma + \mu, b'\sigma + \mu] =$$

$$= [5.8925, 14.1075]$$

NB **L'intervallo di confidenza per la *N* è simmetrico rispetto alla media**, $\mu = 10$

Calcolo della probabilità per la normale con Excel

INV.NORM(probabilità ;media;dev_standard)

Probabilità è la probabilità corrispondente alla distribuzione normale.

Media è la media aritmetica della distribuzione.

Dev_standard è la deviazione standard della distribuzione.

DISTRIB.NORM(x;media;dev_standard;cumulativo)

X è il valore per il quale si desidera la distribuzione.

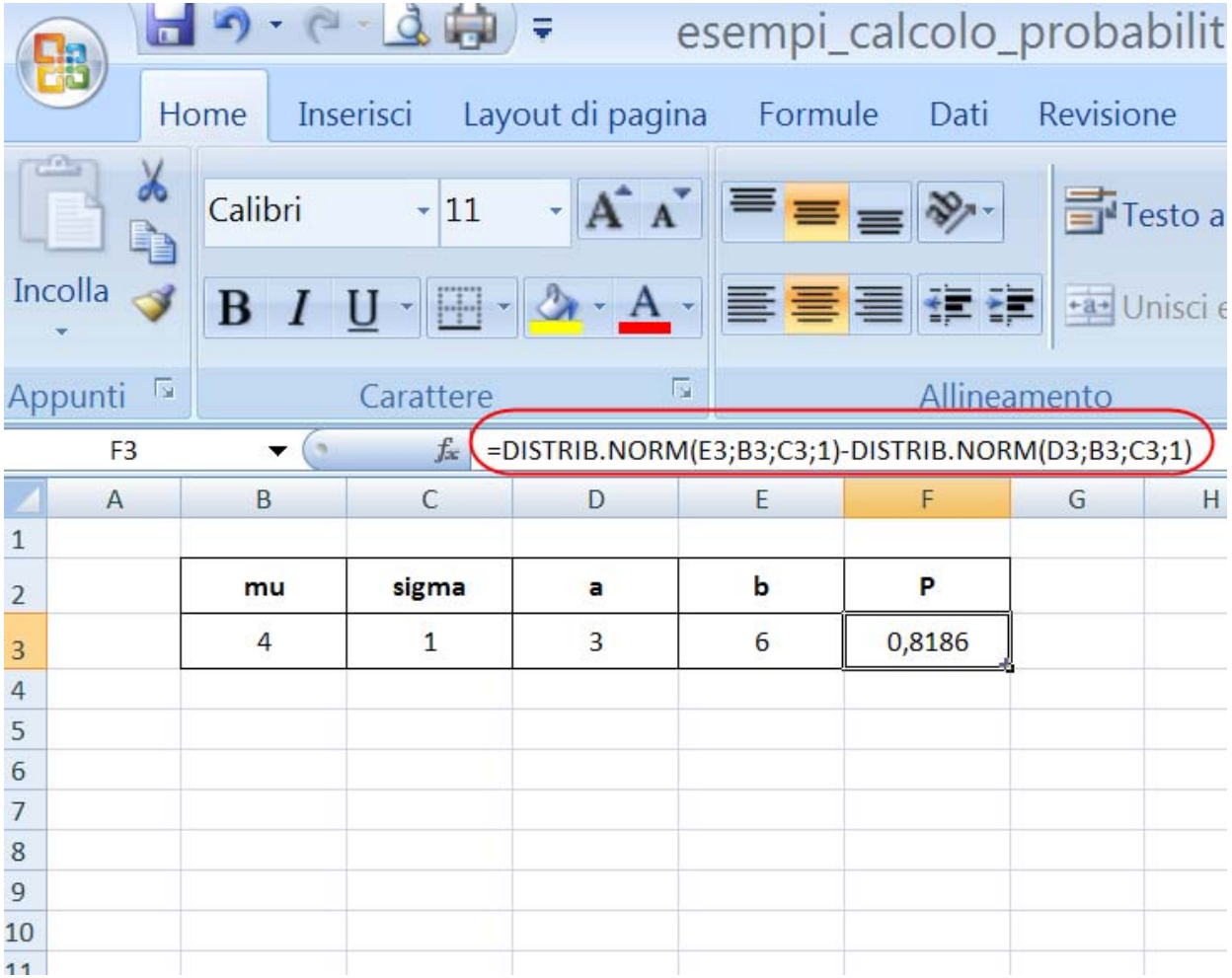
Media è la media aritmetica della distribuzione.

Dev_standard è la deviazione standard della distribuzione.

Cumulativo è un valore logico che determina la forma assunta dalla funzione. Se cumulativo è VERO, DISTRIB.NORM restituirà la funzione distribuzione cumulativa, se è FALSO restituirà la funzione massa di probabilità.

Statistica con Excel - 2

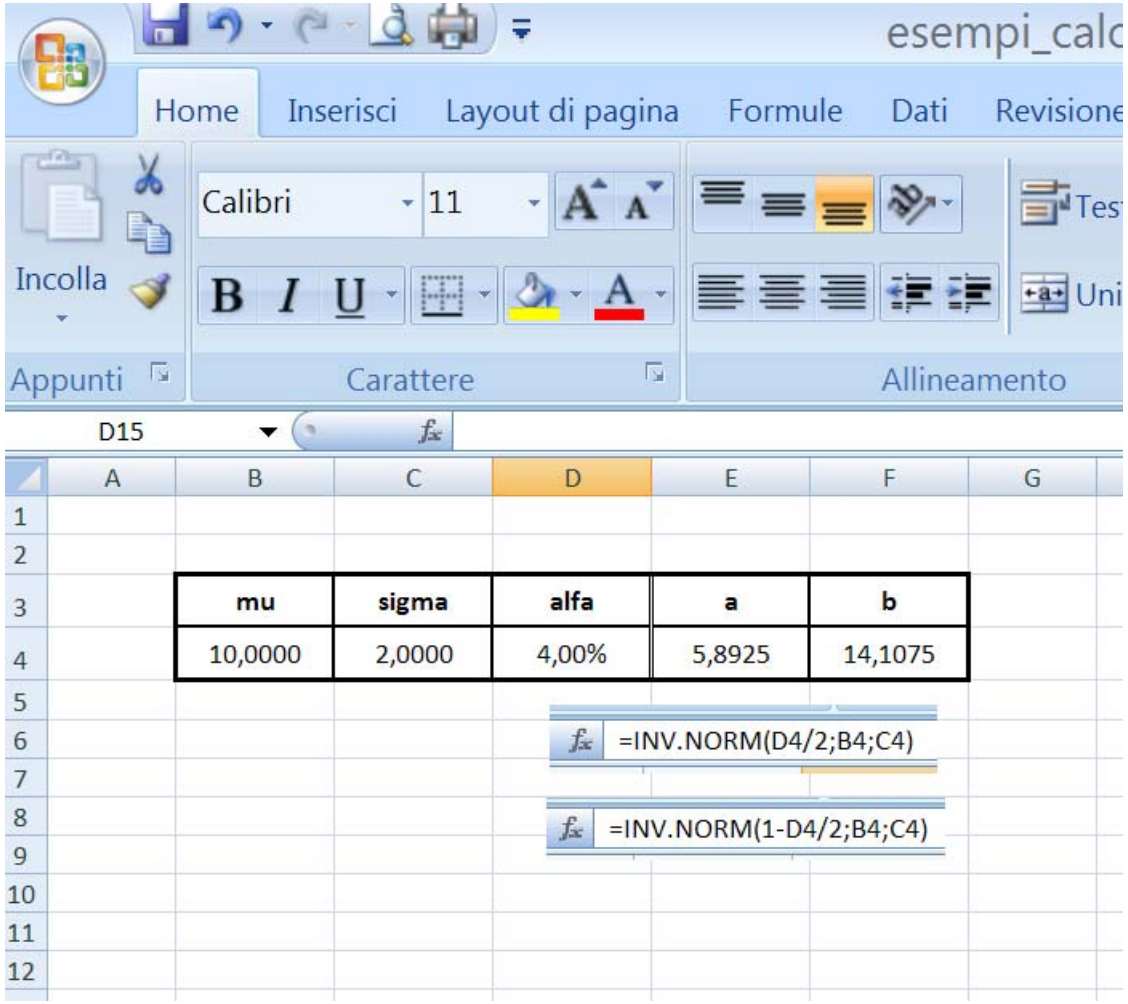
Calcolo della probabilità per la normale con Excel



[esempi_calcolo_probabilita_normale_excel.xlsx; calcolo_prob_normale_excel.png]

Statistica con Excel - 3

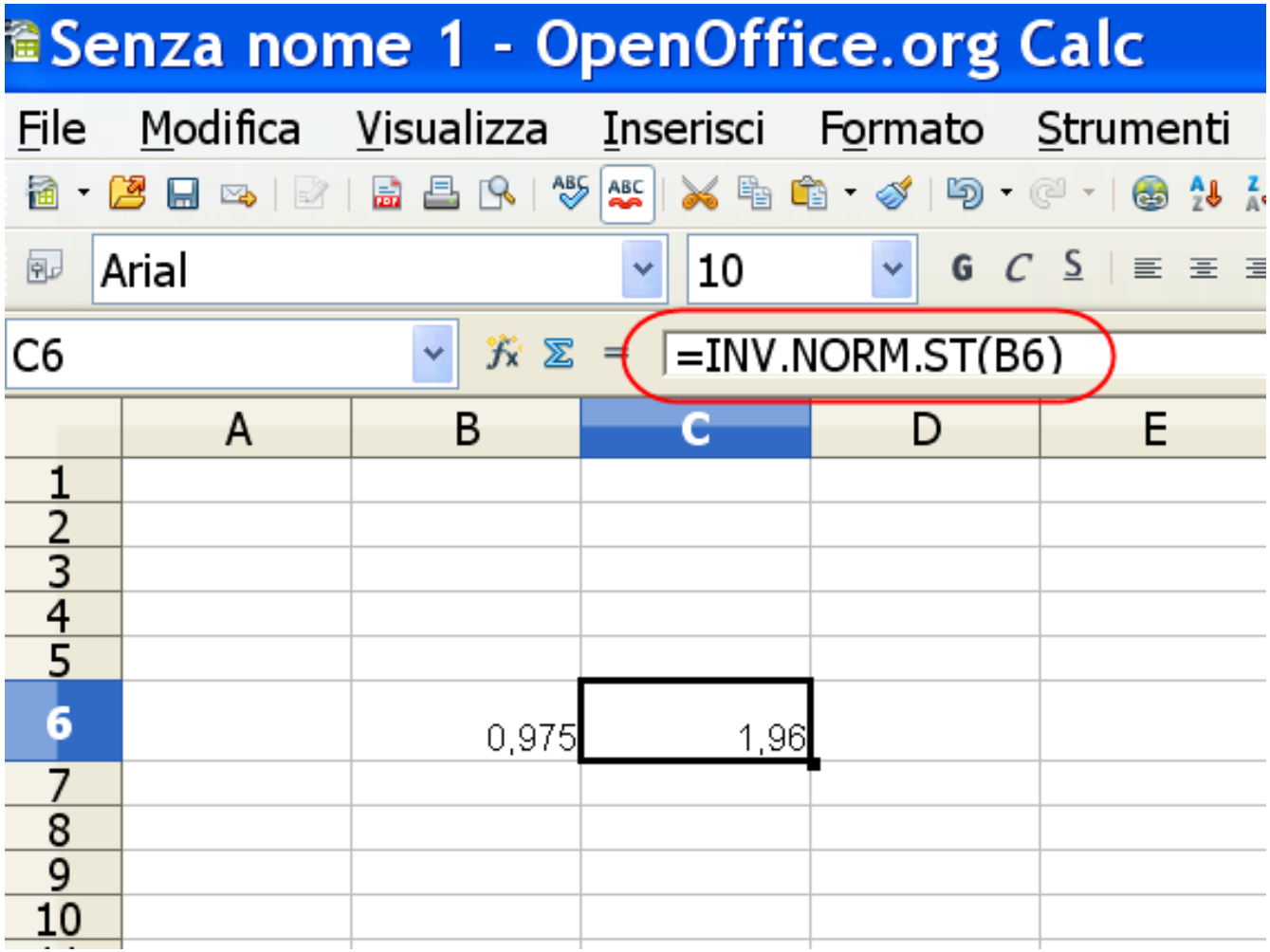
Calcolo di intervalli di confidenza per la normale con Excel



[esempi_calcolo_probabilita_normale_excel.xlsx; calcolo_prob_normale_excel_2.png]

Statistica con OpenOffice CALC

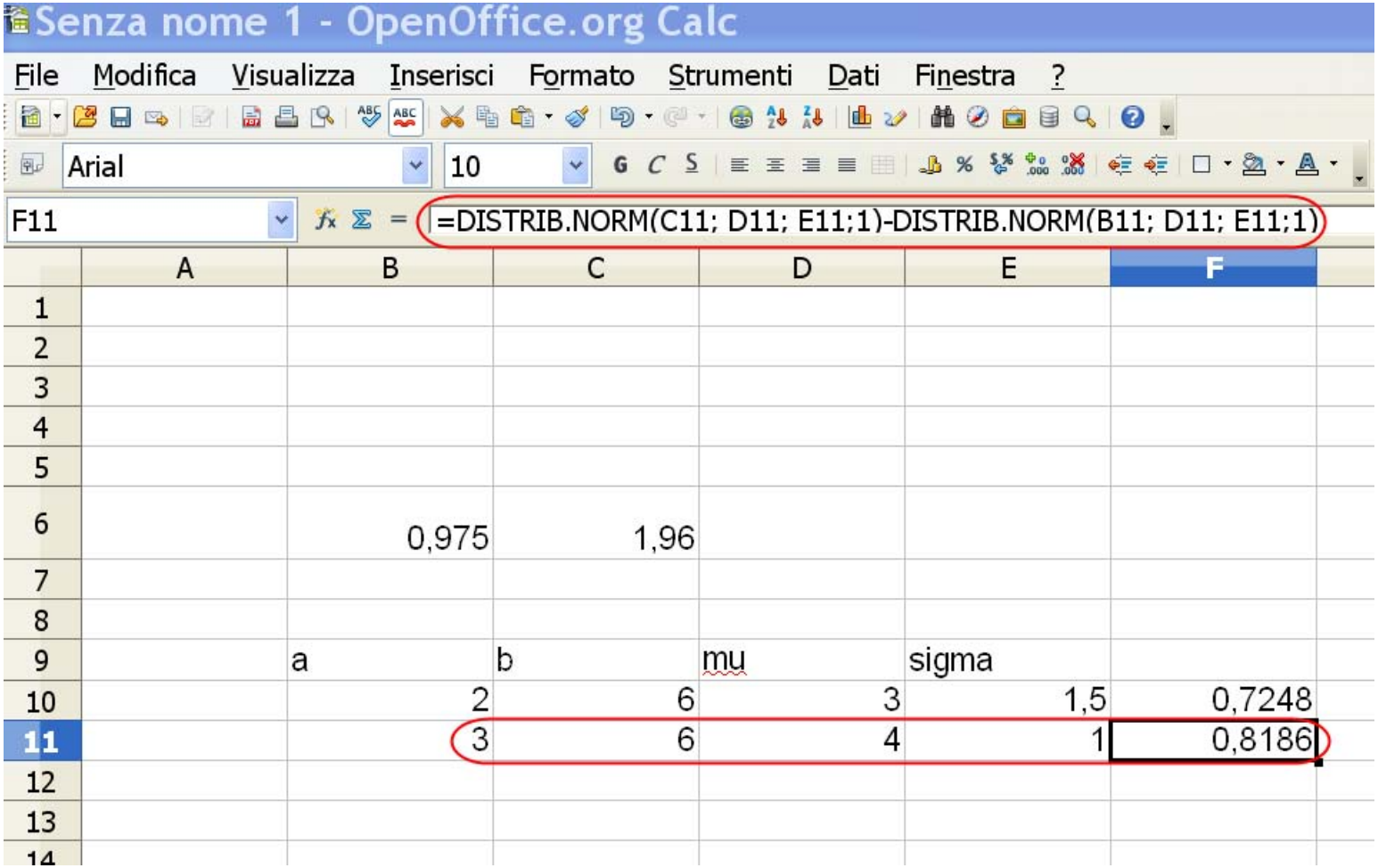
Per calcolare l'intervallo di confidenza della *Z*



[statistica_oocalc_1.png; esempi_statistica_oocalc.ods]

Statistica con OpenOffice CALC - 2

Probabilità di un intervallo per la normale

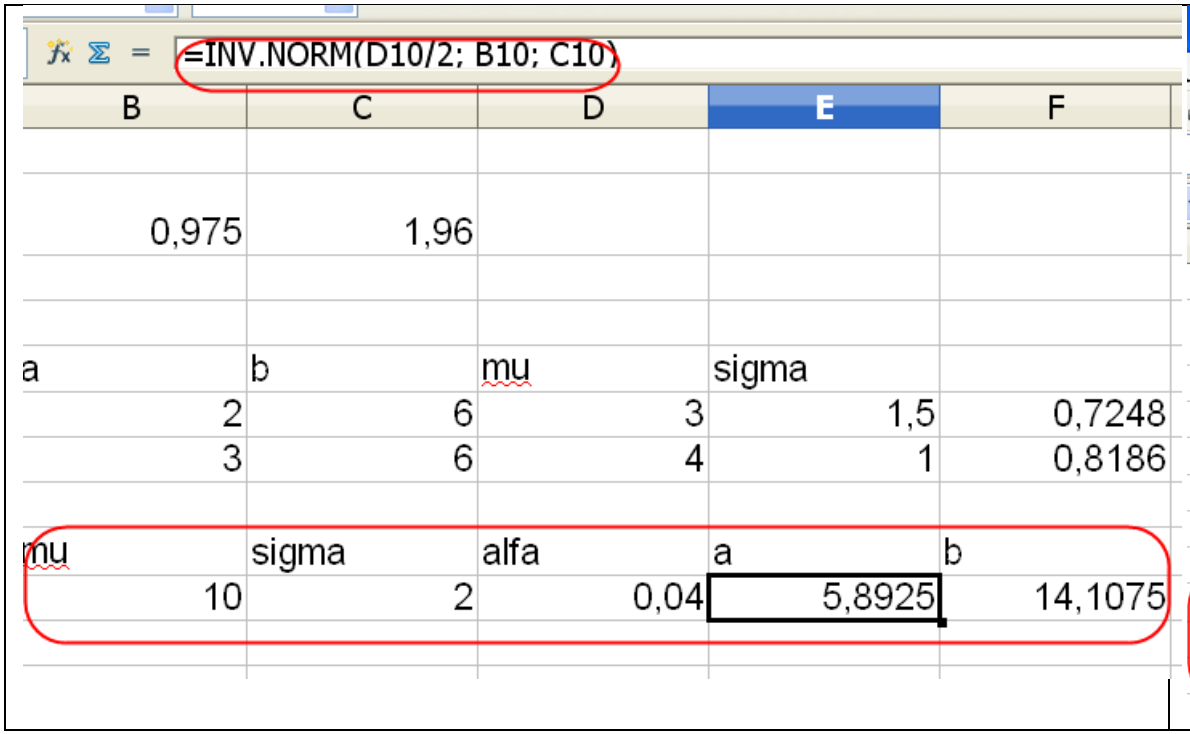


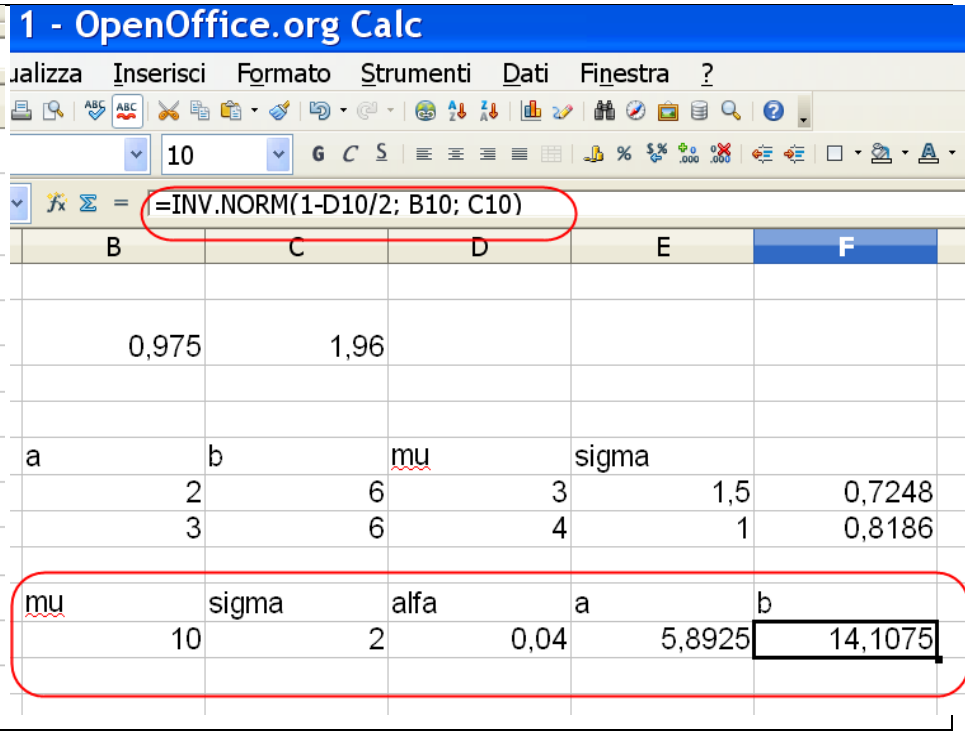
	A	B	C	D	E	F
1						
2						
3						
4						
5						
6		0,975	1,96			
7						
8						
9		a	b	mu	sigma	
10		2	6	3	1,5	0,7248
11		3	6	4	1	0,8186
12						
13						
14						

[statistica_oocalc_2.png; esempi_statistica_oocalc.ods]

Statistica con OpenOffice CALC - 3

Intervallo di confidenza della normale qualunque





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